Constitutive Model of Tendon Responses to Multiple Cyclic Demands (II) - Theory and Comparison -

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The hereditary integral form of a quasi-linear viscoelastic law has been employed. Four new concepts have been employed: 1. a reduced relaxation function with a non-linear exponential function of time, 2. an inverse method to determine the scale factor of the elastic response, 3. an instant elastic recovery strain during unloading, and 4. the results of a constitutive model for cyclic tests may be a function of the Heavyside class. These concepts have been supported by agreement between measured and predicted responses of soft connective tissue to three types of multiple cyclic tests which include rest periods of no extension and alternations between different strain levels. Such agreement has not been attained in the previous studies. Chun and Hubbard (2001) is our companion experimental analysis paper.

Key Words: Hereditary Integral, Viscoelastic Law, Inverse Method, Heavyside Class, Soft Connective Tissue

1. Introduction

To better understand the mechanical responses of collagenous tissues, many researchers have developed constitutive models and compared their predicted results to measured responses from mechanical tests (Fung, 1967; Haut and Little, 1972; Simon, et al 1984; Woo, et al 1981). Fung, 1967 proposed the use of a quasi- linear viscoelastic theory and showed that mechanical responses of rabbit mesentery agreed with his theory using an exponential reduced relaxation function with a standard linear solid viscoelastic model.

Haut and Little, 1972 studied the viscoelastic properties of rat tail tendons. They introduced a logarithmic expression for the reduced relaxation function and a second order stress-strain law. They found that their model was adequate to describe the responses at different strain rates. However, in the case of cyclic extension, the model did not agree well with the experimental data. Woo, et al., 1981 utilized Fung's approach to model the medial collateral ligament. Although agreement between model and experimental data was generally good for a few extension cycles, Woo's model began to predict higher peak and valley stresses than the experimental data within ten cycles.

Lanir, 1980 assumed that the non-linear response of the tissue is due to the waviness of the collagen fibers. He developed a micro-structual model which utilized a function of the distribution of fiber slack lengths. He assumed the collagen fibers were linear viscoelastic in the form of the standard linear solid with an exponential reduced relaxation function. His model predicted that there was less stress relaxation at a lower strain level than at a higher strain level.

None of the currently available models have

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ever been shown to adequately predict the responses to the simple repeated cyclic extension. Yet, many musculoskeletal activities are repeated many times. Thus, a knowledge of the responses of connective tissues to repeated demands is essential to an understanding of their biomechanical roles.

The fundamental questions addressed in this research are :

1. Can a mathematical model be developed to predict the mechanical responses for multiple cyclic extensions?
2. Can a model describe or predict the responses of tissues to extension histories that contain rest periods?
3. Can a model predict the response to a specific strain level of cyclic extensions affected by the previous extensions at a different strain level?

2. Constitutive Model

2.1 Development of constitutive model

For many different biological tissues, a hereditary integral form of the stress-strain constitutive law has been used (Fung, 1967; Haut and Little 1972; Lanir, 1985; Lanir, 1980; Simon, et al, 1984; Woo, et al, 1981). This type of equation, the quasi-linear viscoelastric law (QVL), has been proposed by Fung, 1967 in the form:

$$\sigma(t)=\int_{-\infty}^{t} G(t-\tau)\frac{d\sigma^e[\varepsilon(\tau)]}{d\varepsilon}\frac{d\varepsilon(\tau)}{d\tau}d\tau \quad (1)$$

where $G(t)$ is the reduced relaxation function with $G(0)=1.0$, $\sigma^e(\varepsilon)$ is the elastic response, ε is the strain, t is the current time, and τ is the variable time for integration.

The lower limit of integration was taken to be $-\infty$ in the QVL. Practically, the lower limit should be taken as the origin, $t=0$. If the action starts at time $t=0$, then it is assumed that $\sigma(t)=0$ for $-\infty<t<0$. Also, $\sigma^e=0$ for $t<0$. In other words, the material is completely free of stress and strain initially. Equation (1) may be rewritten as:

$$\sigma(t)=\int_{-\infty}^{0} G(t-\tau)\frac{d\sigma^e[\varepsilon(\tau)]}{d\varepsilon}\frac{d\varepsilon(\tau)}{d\tau}d\tau + \int_{0}^{t} G(t-\tau)\frac{d\sigma^e[\varepsilon(\tau)]}{d\varepsilon}\frac{d\varepsilon(\tau)}{d\tau}d\tau \quad (2)$$

where the first term on the right-hand side of this equation is taken to zero. Thus, we obtain the result in the form:

$$\sigma(t)=\int_{0}^{t} G(t-\tau)\frac{d\sigma^e[\varepsilon(\tau)]}{d\varepsilon}\frac{d\varepsilon(\tau)}{d\tau}d\tau \quad (3)$$

For the reduced relaxation function, an exponential expression for a standard linear solid viscoelastic model, $G(t)=\alpha+\beta e^{-ut}$, and a logarithmic expression, $G(t)=g-h\ln t$, have been used thus far. However, the above expressions have not described well the viscoelastic nature of biological tissues. The logarithmic equation (Haut and Little, 1972; Woo, et al, 1981) yields negative values with large time and is not appropriate for modeling long term responses of soft tissues. Because soft tissues never carry negative values for the relaxation test with large time. The standard linear viscoelastic model (Lanir, 1980; Simon, et al, 1984) does not fit both short and long term responses for soft tissues.

From the analysis of the relaxation tests, a new reduced relaxation function for soft tissues is proposed in the form (Chun, 1986; Chun and Hubbard, 1986) by the authors:

$$G(t)=\alpha+\beta e^{-\mu t^q} \quad (4)$$

We call this form the standard non-linear solid reduced relaxation function, where α, β, μ and q are positive constant values.

Fung, 1967 showed the elastic response for rabbit mesentery was an exponential expression, which may be written as:

$$\sigma^e=A\left[\lambda-\frac{1}{\lambda^2}\right]e^{b\lambda} \quad (5)$$

where A and b are constants.

The Lagrangian strain and the stretch ratio are defined as follows :

$$\lambda=\frac{L}{L_0},$$

$$\varepsilon=\frac{L-L_0}{L_0}=\lambda-1 \quad (6)$$

Thus, Eq. (5) may be rewritten in terms of strain as :

$$\sigma^e = B\left[\varepsilon + (b-1)\varepsilon^2 + \left(\frac{b^2}{2} - b + \frac{4}{3}\right)\varepsilon^3 + \cdots\right] \quad (7)$$

where $B(=3\ Ae^b)$ is constant.

The stress-strain relationship of collagen fibers has led to the choice of the second term of Eq. (7) from a study of rat tail tendons by Haut and Little, 1972. Also, this second order relation has been found by Hubbard and Chun, 1985 in long term cyclic tests of the tendon. The previous studied and the results presented in Part I: Experimental Analysis (Chun and Hubbard, 2001) support the second order term of Eq. (7) to represent the elastic response of tendon.

Rigby, et al., 1959 used photographs taken by transmitted polarized light to show that the waviness of collagen fibers straightened during loading and reappeared upon subsequent unloading. Their strain level had not exceeded 4% in a study of wet rat tail tendons. Viidik, 1973 found in a study of rat tail tendon that, first, the waviness becomes "shallower" during stretching; second, the tendon bundles straightened out completely at the end of the toe part in the stress-strain curve; and, finally, a short period waviness appeared during unloading. Also, Kastelic, 1979 showed the same phenomena in his morphological model for the crimp structure of tendon with a polarizing microscope technique. Lanir, 1980 reported that three factors may contribute to the recrimping of the collagen: the collagen's own bending rigidity, its interaction with the ground substance, and the stretch exerted by the elastin. Also, Lanir, 1985 has reported that:

"The QVL has some restrictions with regards to negative strain rate cases; the tissue response is expected to differ from the QVL owing to the anticipated difference between the viscoelastic responses of stretched vs. contracting crimped fibers in unloading phase of cyclic tests".

Recrimping of the collagen during unloading may contribute some degree of the asymmetry of the stress-strain relationship between the loading and unloading phase. In the present study, the term instant elastic recovery strain is introduced into the QVL to describe the additional asymmetry phenomena which may be caused by recrimping during unloading. Based on previous works (Fung, 1967; Haut and Little, 1972; Hubbard and Chun, 1985; Kastelic, 1979; Lanir, 1985; Lanir, 1980; Rigby, et al, 1959; Viidik, 1973), it is reasonable to assume that the elastic response for the soft tissues for loading and unloading would be in the form:

$$\sigma^e = K_2[\varepsilon(t) + \varepsilon_e]^2 \quad (8)$$

where K_2 is a scale factor of the elastic response, ε_e is the instant elastic recovery strain for unloading (negative strain rate), and $\varepsilon_e = 0$ for nonnegative strain rate. Differentiation of Eq. (8) results in:

$$\frac{d\sigma^e}{d\varepsilon} = 2K_2[\varepsilon(t) + \varepsilon_e] \quad (9)$$

By substituting Eqs. (4) and (9) into Eq. (3), then the constitutive equation is obtained in the form:

$$\sigma(t) = 2K_2\int_0^t [\alpha + \beta e^{-\mu(t-\tau)^q}][\varepsilon(\tau) + \varepsilon_e]\frac{d\varepsilon(\tau)}{d\tau}d\tau \quad (10)$$

2.2 Relaxation Response

In the stress relaxation test under constant strain level, the strain function may be written in the form:

$$\varepsilon = \varepsilon_0 U(t) \quad (11)$$

where $U(t)$ is the unit step function (Abramowits and Stegun, 1964). Differentiation of the Eq. (11) yields:

$$\frac{d\varepsilon}{dt} = \varepsilon_0 \delta(t) \quad (12)$$

where $\delta(t)$ is the Dirac delta function.

By substituting Eqs. (11) and (12) into Eq. (10) and setting $\varepsilon_e = 0$ (since this is a nonnegative strain rate test), then the constitutive equation yields:

$$\sigma(t) = 2K_2\int_0^t [\alpha + \beta e^{-\mu(t-\tau)^q}]\varepsilon_0 U(\tau)\varepsilon_0\delta(\tau)d\tau \quad (13)$$

And, Eq. (13) is simply integrated by the definition of the Dira delta function (Abramowits and Stegun, 1964). The result of this integration is:

$$\sigma(t) = K_2\varepsilon_0^2[\alpha + \beta^{-\mu t^q}] \quad (14)$$

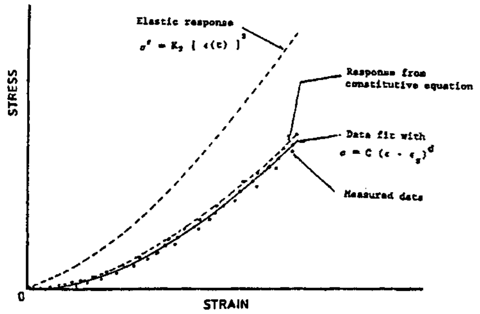

Fig. 1 Illustration of relationship between measured data, the regression fit (with C and d), the elastic response (with K_2 and m), and response from the constitutive equation

which is the response to a stress relaxation test.

2.3 Response in the constant strain rate loading test

In the constant strain rate loading test, the strain function may be written in the form:

$$\varepsilon(t)=\gamma t \tag{15}$$

where γ is the constant strain rate. Differentiation of Eq. (15) yields:

$$\frac{d\varepsilon(t)}{dt}=\gamma \tag{16}$$

By substituting Eqs. (15) and (16) into Eq. (10) and setting $\varepsilon_e=0$ (since this is nonnegative strain rate test), then the constitutive equation yields:

$$\sigma(t)=2K_2\gamma^2\int_0^t[\alpha+\beta e^{-\mu(t-\tau)^q}]\tau d\tau \tag{17}$$

which is the response to a constant strain rate loading test.

2.4 Response in the constant strain rate multiple cyclic test

In the constant strain rate multiple cyclic tests (see Fig. 1 in Part I : Experimental Analysis (Chun and Hubbard, 2001), the strain function may be written in the form:

$$\varepsilon(t)=(-1)^{n+1}\gamma(t-XINT) \tag{18}$$

where n is the half cycle counting integer, γ is the constant strain rate, and XINT is the time for zero strain in the current half cycle. XINT occurs at the beginning of each loading half cycle and at the end of each unloading half cycle. Differentiation of Eq. (18) yields:

$$\frac{d\varepsilon(t)}{dt}=(-1)^{n+1}\gamma \tag{19}$$

In the constant strain rate cyclic tests, we shall assume the following for the instant elastic recovery strain:

$$\varepsilon_e=\gamma\varepsilon_K \tag{20}$$

where ε_K, the fraction of the instant elastic recovery strain, is given by ε_K=(first cycle unloading slack strain-first cycle loading slack strain)/N, and N is the full cycle counting integer ($=n/2$).

The loading slack strain for the first cycle of an experiment is usually adjusted to be zero so that the value of ε_K is equal to the unloading slack strain in the first cycle. This ε_K value decreases as the number of cycles (N) increases.

Substitution of Eqs. (18), (19) and (20) into Eq. (10) yields:

$$\sigma(t)=(-1)^{n+1}2K_2\gamma^2\int_0^t[\alpha+\beta e^{-u(t-\tau)^q}] [(-1)^{n+1}(\tau-XINT)+\varepsilon_K]d\tau \tag{21}$$

which is the response to a constant strain rate multiple cyclic test.

In the constitutive Eq. (21), there are numerical values which are based on measurements of tendon responses. The reduced relaxation function incorporates α=0.27, β=0.73, μ=0.59 and q=0.129 and their numerical values have been found (Chun, 1986; Chun and Hubbard, 1986), which are average values for 3% peak strain level. The elastic response incorporates the exponent m=2, the scale factor (K_2), and the instant elastic recovery strain (ε_K). The instant elastic recovery strain is based on a value of unloading slack strain in the first extension cycle, a representative value of this slack strain has been determined to be 0.85%. The values of the scale factor (K_2) will be examined in the next.

The result of the hereditary integral form of a constitutive equation for cyclic tests may be a function of the Heavyside class. The negative values are retained in the modeling calculations. For presentation, however, the negative values from this modeling calculation are set identically to zero, because the tendon does not transmit

Table 1 Comparison between regression fit coefficients (C, d) from the measured data and coefficients (K_2, $m=2$) of the elastic response at the first loading for 3% strain level tests. Multiple cyclic test types (A, B, C) were determined in Part I : Experimental Analysis (Chun and Hubbard, 2001)

Test Type-No.	C(Gpa)	*d*	K_2(Gpa)	*m*
A-1	11.66	1.68	47.40	2
A-2	27.30	1.94	44.41	2
A-3	24.20	2.17	17.41	2
A-4	26.58	2.26	14.40	2
A-5	11.51	1.83	27.67	2
A-6	13.52	2.03	16.22	2
B-1	7.42	1.67	31.14	2
B-2	47.01	2.15	35.75	2
B-3	14.69	1.97	21.57	2
B-4	13.48	1.87	28.62	2
B-5	4.10	2.00	5.34	2
B-6	64.01	2.42	20.26	2
C-1	12.35	1.99	16.93	2
C-2	25.71	2.15	19.77	2
C-3	28.84	2.21	18.09	2
C-4	33.08	2.31	15.14	2
C-5	0.86	1.74	2.90	2
C-6	26.66	2.48	6.69	2
Ave.		2.05		2
S.D.		0.24		0
95% C.I.		0.12		0
Max.	64.01		47.40	
Median	19.44		18.93	
Min.	0.86		2.90	

longitudinal compressive stress (load) during unloading in the cyclic test.

3. Inverse Method to Determine the Scale Factor K_2

The constitutive model (Eq. 10) incorporates the reduced relaxation function (Eq. 4) and the elastic response (Eq. 8). Within each of these functions are numerical coefficients whose values are based on measured data. Except for K_2 in the elastic response, representative values of all these coefficient values have been determined.

K_2 is a scale factor in the second order elastic stress-strain relationship which is analogous to a modulus of elasticity in a linear stress-strain relation. Thus far, many authors (Fung, 1967; Haut and Little, 1972; Lanir, 1980; Simon, et al, 1984; Woo, et al, 1981) have used a curve fitting equation from sufficient high strain rate tests for the elastic response. However, relaxation is a part of any measurement of tendon responses from any kind of strain rate tests so that the elastic response cannot be measured directly. In other words, the elastic response can only be approached as the strain rate of an extension test approached infinity. This is not practical. Thus, determination of K_2 from measured data requires use of the constitutive model which combines the elastic and viscous responses of the tendons.

In the constant strain rate tests, the instant elastic recovery strain (ε_e) has been defined as zero. Thus, Eq. (8) may be written as:

$$\sigma^e = K_2[\varepsilon(t)]^2 \tag{23}$$

The K_2 may be determined using the constitutive equation for a constant strain rate extension expressed in Eq. (17):

$$\sigma(t)=2K_2\gamma^2\int_0^t[\alpha+\beta e^{-\mu t^q}]\tau d\tau \qquad (24)$$

At $t=t_1$ (the time at peak strain level),

$$\sigma(t)=2K_2\gamma^2\int_0^{t_1}[\alpha+\beta e^{-\mu t^q}]\tau d\tau \qquad (25)$$

Finally,

$$K_2=\frac{\sigma(t_1)}{2\gamma^2\int_0^{t_1}[\alpha+\beta e^{-\mu t^q}]\tau d\tau} \qquad (26)$$

where $\sigma(t_1)$ is the peak stress measured from when the strain level reaches the first peak ($t=t_1$).

Figure 1 is an illustration of the relationships between measured data, the regression fit (with C and d) from the measured data which already includes viscous phenomena, the elastic response (with K_2 and $m=2$) which does not include any viscous phenomena, and the response from the constitutive equation which combines both an elastic response and a reduced relaxation function.

Table 1 compares the coefficients C and d with the coefficients K_2 and m for 5%/sec constant strain rate tests. In this table, the median value of K_2 is similar to the median value of C and the average value of d is similar to the value of $m=2$. However, in each test, the value of K_2 differs greatly from the value of C. These differences are due to the interactions between C and d, which are both variables determined from regression of the measured data. Also, K_2 is determined with $m=2$, and K_2 is not directly comparable to C which is determined by a regression fit to measured response as shown in Fig. 1.

Scatter in the values of K_2 was too great to choose an average value of K_2 for general use. This scatter indicates that the value of K_2 for a specific tendon should be chosen from that tendon's test result as a scale factor of the elastic response.

4. Comparison Between Predicted and Measured Results

Knowledge of tendon responses gained from the measured results in Part I : Experimental Analysis (Chun and Hubbard, 2001) have influenced the modeling assumptions in this paper. Values of the numerical quantities in the constitutive model have been selected to be representative of measured results. Variability of numerical values in the reduced relaxation function was small enough so that the average values from the 3% constant peak strain relaxation tests are used in representing the relaxation response of all the tendon specimens. As also described in the previous chapter, these relaxation coefficients are: $\alpha=0.27$, $\beta=0.73$, $\mu=0.59$, and $q=0.129$. In the elastic response, a second order function of strain ($m=2$) was chosen to be representative of all results. Also, there was small variability in the values of the unloading slack strain in the first extension cycle of the multiple cyclic tests. Thus, the average value of this unloading slack strain (0.85%) was used in the calculation of the instant elastic recovery strain in the model. Because of large variability in values between specimens, the scale factor, K_2, of the elastic response must be chosen for each specimen.

The multiple cyclic tests involved three different type (A, B, and C-type) of cyclic test sequences (see Fig. 1 in Part I : Experimental Analysis (Chun and Hubbard, 2001). In the comparison of results from the constitutive model with measured data in the multiple cyclic tests, predicted and measured results will be presented for selected specimens from Part I : Experimental Analysis (Chun and Hubbard, 2001) and then predicted results will be compared with measured results which have been statistically summarized.

4.1 A-type multiple cyclic test

Figure 2 shows the cyclic stress relaxation for the test A-1. The predicted results were obtained from the Eq. (21) with a value of $K_2=16.22$GPa. The peak stresses from the model agree very well with the peak stresses from the measured data throughout the entire 300 cycles of the test.

Figures 3 through 7 show the stress-strain plots for the test A-2(from the first cycle to 300^{th} cycle) with a value of $K_2=44.42$ GPa. In these figures, the agreement between the predicted and measured results for the hysteresis loops is good

Table 2 Comparison of normalized cyclic stress relaxation between predicted and average measured data for A-type tests

Cycle No. Time	1	2	3	60 72s	120 144s	180 216s	240 288s	300 360s
Predicted	100.0	95.9	94.0	83.1	81.2	80.1	79.2	78.7
Measured Ave.	100.0	96.7	95.2	85.9	83.3	82.0	80.5	79.5
95% C.I.	0.0	0.9	0.8	1.8	2.1	2.5	2.2	2.0

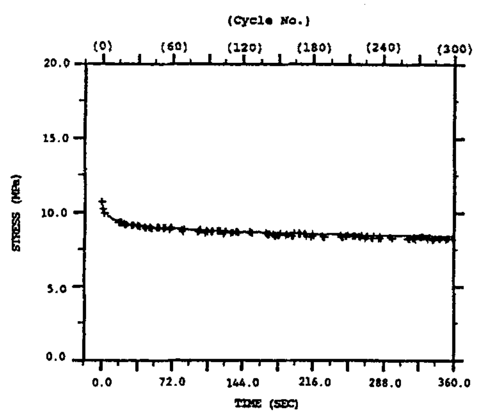

Fig. 2 Cyclic stress relaxation for test A-6 with 5%/sec constant strain rate (-----: Predicted data, ++++: Measured data)

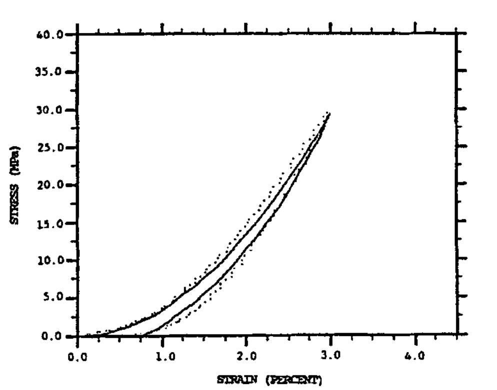

Fig. 3 Stress-strain plot for the 1^{st} cycle in test A-2 with 5%/sec strain rate (—— : Predicted data, ····: Measured data)

in the first three cycles but the predicted results start to deviate a little from the measured data with successive cycles.

To summarize the measured peak stress values from multiple cyclic tests, it was necessary to normalize these values to the first peak with a value of 100. This normalization removed most of the variance in the measured peak stresses from specimen to specimen. For comparison with the measured results, the predicted results have also been normalized. The effects of different K_2 values are removed by this normalization so that predictions are the same for all specimens within a test type.

Normalized peak stress values predicted from the model are presented in Table 2 with the averages and 95% confidence intervals for measured results from Table 2 in Part I: Experimental Analysis (Chun and Hubbard, 2001). The predicted values are nearly identical to the average measured values and close to or within the 95% confidence interval throughout the entire 300 cycles.

This close agreement of predicted and measured normalized peak stress values is remarkable since these predictions are based on numerical input to the model which is representative of all tendons tested. Such close agreement in the peak stress values has never before been attained for cyclic responses of this duration. The qualitative agreement indicated that the modeling assumptions are basically sound for predicting response in this type of test.

4.2 B-type multiple cyclic test

Figure 8 shows the measured and predicted cyclic stress relaxation for the test B-3. Predicted results were obtained from the Eq. (21) with the parameter K_2=21.57GPa, and the other parameter values were the same as the parameter values for other modeling. As in the A-type test, the agreement in the initial 60 cycles is very good. The recovery in both predicted and measured results after each 72 seconds rest period at 144 sec (61^{st} cycle) and 288 sec (121^{st} cycle) are nearly the same. The predicted peak stresses (relaxation and recovery) from the model agree very well

Table 3 Comparison of normalized cyclic stress relaxation between predicted and average measured data at B-type tests

Cycle No. Time	1	2	3	60 72s	61 144s	120 216s	121 288s	180 360s
Predicted	100.0	95.9	94.0	83.2	89.7	81.9	88.3	81.1
Measured Ave.	100.0	96.2	94.2	83.2	86.1	80.9	83.4	79.0
95% C.I.	0.0	1.5	3.5	7.5	7.7	8.9	8.9	9.6

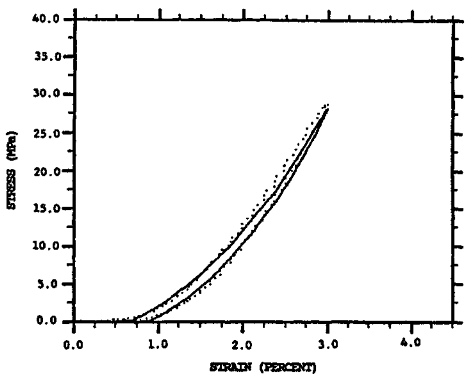

Fig. 4 Stress-strain plot for the 2nd cycle in test A-2 with 5%/sec strain rate (—— : Predicted data, ····: Measured data)

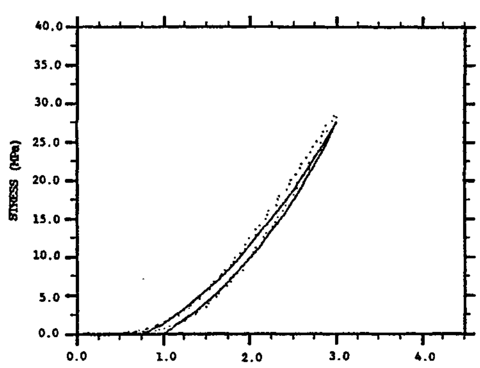

Fig. 5 Stress-strain plot for the 3rd cycle in test A-2 with 5%/sec strain rate (—— : Predicted data, ····: Measured data)

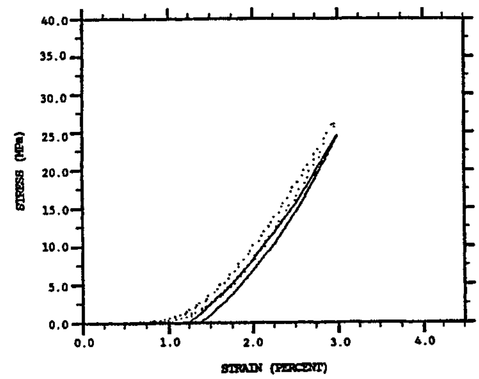

Fig. 6 Stress-strain plot for the 60th cycle in test A-2 with 5%/sec strain rate (—— : Predicted data, ····: Measured data)

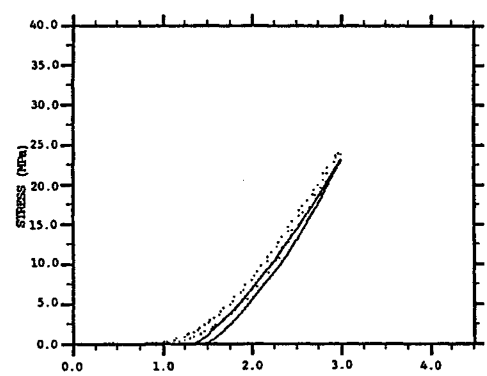

Fig. 7 Stress-strain plot for the 300th cycle in test A-2 with 5%/sec strain rate (—— : Predicted data, ····: Measured data)

with the measured peak stresses throughout the entire 180 cycles.

Normalized peak stress values predicted from the model for a B-type multiple cyclic test are presented in Table 3 with comparable measured results (averages and 95% confidence intervals) from Table 3 in Part I : Experimental Analysis (Chun and Hubbard, 2001). The agreement for the first 60 cycles is almost perfect. After the first 72 seconds rest period, the model predicts an increase of peak stress (recovery) which is slightly greater than measured average but within the 95% confidence interval. By the end of the second cyclic block (at 120st cycle), the predicted and measured values are nearly the same. This slightly higher predicted recovery followed by a return to

Table 4 Comparison of normalized cyclic relaxation between predicted and average measured data for C-type tests

Cyc No.	1	2	3	60	61	105	106	165	166	210	211	270
Time				72s		144s		216s		288s		360s
PKSL	3	3	3	3	4	4	3	3	4	4	3	3
Predicted	100.0	95.9	94.0	83.2	159.4	152.1	75.6	78.9	155.2	149.6	73.1	77.1
M. Ave.	100.0	94.5	91.7	77.3	189.1	162.2	54.8	56.8	166.0	156.4	50.0	51.9
95% CI	0.0	2.6	3.9	8.8	27.3	19.0	15.7	15.2	19.7	19.0	16.6	16.0

Cyc No. : Cycle Number, PKSL: Peak Strain Level (%), M. Ave. : Measured Average
95% CI: 95% Confidence Interval

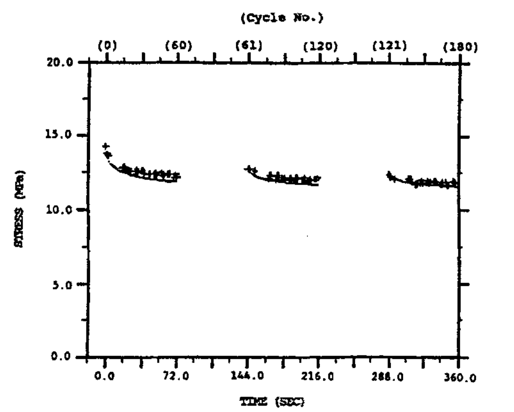

Fig. 8 Cyclic stress relaxation for test B-3 with 5%/sec strain rate (----- : Predicted data, ++++ : Measured data)

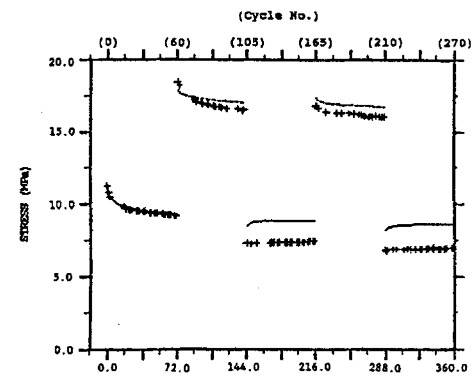

Fig. 9 Cyclic stress relaxation for test C-1 with 5%/sec strain rate (----- : Predicted data, ++++ : Measured data)

measured responses is repeated in the third cyclic block from the 121^{st} cycle to the 180^{st} cycle. As with the A-type test, the close agreement between measured and predicted responses is remarkable.

4.3 C-type multiple cyclic test

Figure 9 shows the measured and predicted cyclic stress relaxation for the test C-1. Predicted results were obtained from the Eq. (21) with the paremeter K_2=16.93GPa, and the other parameter values were the same as the parameter values for the other modeling. In the first cyclic block at 3% peak strain (0 to 72 sec), the predicted peak stresses agree very well with the measured peak stresses as in the previous cases. When the peak strain increases from 3% to 4% at the beginning of the second cyclic block (61^{st} cycle at 72 sec), measured values are slightly below the predicted value. During the second cyclic block, both relax with cydes and the measured values decreasing more rapidly to a value below the predicted value by the 105^{th} cycle at 144 sec. With the return to 3% peak strain in the third cyclic block, both peak stress values decrease to about half their previous values with the measured value below the predicted value. Then the model predicts the increase (recovery) of measured peak stress. When the peak strain returns to 4%, the stresses about double again to nearly the same values as in the second cyclic block. They then relax until the end of that cyclic block. As the peak strain returns to 3%, the stresses drop by about half their previous values with the predicted values slightly greater than the measured values. In the final cyclic block, both predicted and measured stresses increase (recover) with the predicted values increasing more.

Normalized peak stress values predicted from the model for a C-type multiple cyclic test are presented in Table 4 with comparable measured results (averages and 95% confidence intervals) from Table 4 in Part I : Experimental Analysis (Chun and Hubbard, 2001). The pattern of response in this test type has been discussed above with reference to Fig. 9. After the first cyclic block, the average values of measured peak stress is consistently above the predicted values in the cycles to a peak strain of 4% and below the predicted values in the cycles to a peak strain of 3%. Also, the predicted values are near the extremes or beyond the confidence intervals of the measured values. The confidence intervals for the C-type test are larger than the other type of tests.

The quantitative agreement between predicted and measured peak stresses in the C-type test is not as exact as in the previous type tests (A and B-type). Yet, the model does predict the responses for the C-type test which are within or close to the 95% confidence intervals and the qualitative agreement is good.

5. Conclusion

The work reported here is a study to model and compare tendon responses to multiple cyclic tests including constant peak strain level cyclic test (A-type), constant peak strain level cyclic test with two rest periods (B-type), and different peak strain level cyclic test (C-type).

In the constitutive modeling, the hereditary integral form of a quasi-linear viscoelastic law has been used with four new features. First, a nonlinear exponential reduced relaxation function was developed and employed for the time dependent part. Second, a new concept of elastic response with instant elastic recovery effect during unloading was developed and employed for the strain dependent part. Third, an inverse method was developed to determine the scale factor of the elastic response. Fourth, the results of a constitutive model for cyclic tests might be a function of the Heavyside class.

The comparisons between predicted results and measured data have been made for examples and averaged results with 95% confidence intervals from all types of tests including three different type (A, B, and C) of multiple cyclic tests. The value of K_2 was determined from the measured peak stress of the first extension at the each test and the constitutive equation. Agreement between measured data and the calculated results from the constitutive model was very close. Thus, this coincidence of the measured and calculated peak stress in the first extension is to be expected, but the close agreement throughout the entire curve up to the peak stress does support the predictive capability of the constitutive model.

In the present study, substantial progress has been made in modeling viscoelastic responses of tendons in diverse and complex situations. The good agreement between measured and predicted responses contribute to a better understanding of musculoskeletal performance in activities of daily living, athletic performance, and manipulative therapy.

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